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FUZZY DESIRABILITY EVALUATION STRUCTURE FOR MULTI-RESPONSE INFERENCE SYSTEM OPTIMIZATION USING THE GENETIC ALGORITHM

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ABSTRACT

Fuzzy rule-based systems are among the best techniques for modeling and solving real-world complex problems which include plenty of inputs and outputs. Emulating the reasoning process of a human expert is the main characteristic of these systems. Rule-based systems are also appropriate for other purposes including consulting, diagnosis, learning, decision support, design, planning, or research. The present study intends to find optimized solutions for multi-response problems having multiple conflicting objectives using a fuzzy inference method. The fuzzy outputs of the considered type of problems are evaluated here applying a proposed desirability mapping structure tailored for fuzzy responses. Since ordinary desirability functions do not apply to fuzzy output variables according to the different possible cases, a customized desirability evaluation structure and defuzzification technique is proposed in this regard. Additionally, the genetic algorithm is applied to search among the input values that optimize the whole responses simultaneously. Eventually, the application of the model is described in a numerical example.

KEYWORDS: Multi-Response Optimization; Multi-Criteria Decision-Making; Fuzzy Inference System; Fuzzy Expert System; Genetic Algorithm

1. INTRODUCTION

In a typical optimization problem, the purpose of the decision maker is to specify a feasible set of input conditions (known as independent variables), which leads to a satisfactory output (known as response variable) (Pasandideh & Niaki, 2006). Response surface methodology (RSM) is one of the most common techniques that accomplish such purposes. RSM is a collection of mathematical and statistical techniques that are useful for the modeling and analysis of problems in which a response of interest is influenced by several variables and the objective is to optimize this response (Baş & Boyacı, 2007).

In real-world problems, we often aim to optimize several response variables simultaneously and this is the point where the solution starts to get complicated. This complication occurs because the selected levels of inputs that optimize one output might not even come close to optimizing another one (Pasandideh & Niaki, 2006). Also, there are some techniques suggested by RSM to optimize multiple responses that work well when there are only a few input variables. However, when there are more than three designs or independent variables, RSM faces some problems, and this is because of the visual restrictions of plotting multi-dimensional graphs. This

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problem can be solved via the allocation of constant values to not all but two input variables, however, this is not a practical way to reliably optimize problems with many input variables (Amiri et al., 2009).

Therefore, there is a practical interest in more formal optimization methods for multiple responses. A popular approach is to formulate and solve the problem as a constrained optimization problem. To solve this problem, many numerical techniques exist which sometimes are referred to as nonlinear programming methods.

Implementation of optimization processes on mathematical programming problems or simulated systems has become an ordinary issue on which comprehensive studies have been conducted. Nevertheless, optimization of other types of functional mapping of systems could be important in engineering problems. The inference engine of a fuzzy expert system (FES) can be referred to as one of these mappings (Bojadziev & Bojadziev, 2007).

An expert system is a computer-based system that emulates the reasoning process of a human expert within a specific domain of knowledge (Tavana & Hajipour, 2019). Expert systems are primarily built to make the experience, understanding, and problem-solving capabilities of the expert in a particular subject area available to the non-expert in that area (Liao et al., 2013; Tzouvaras et al., 2005). In addition, they may be designed for various specific activities, such as consulting, diagnosis, learning, decision support, design, planning, or research (Peeva & Kyosev, 2004). Also, the inference engine of a fuzzy expert system (known as a fuzzy rule-based system) operates on a series of production rules and makes fuzzy inferences (Ramadoss & Krishnaswamy, 2015).

Exact optimization processes applied in nonlinear programming methods are quite useful for simple problems with few input and output variables. However, these techniques are not useful for complex real-world problems having plenty of inputs and outputs. Fuzzy rule-based systems are good candidates for modeling and solving such problems (Bojadziev & Bojadziev, 2007).

In such situations, heuristic or meta-heuristic algorithms could be successfully applied to obtain feasible solutions. These algorithms do not optimize the problem exactly but provide answers which are adjacent to the optimum (Yaghoubzadeh-Bavandpour et al., 2022). The genetic algorithm, which is used in this article, is one of these meta-heuristic algorithms.

Optimization of responses is performable based on different decision-making preferences for which the desirability evaluation structure is considered a generic approach (Akteke-Ozturk et al., 2018). Presenting a fuzzy desirability evaluation structure, its alignment with a fuzzy inference system, and applying the genetic algorithm to search for optimum solutions in the model are triple contributions of the current study. These concepts obtain additional value in particular when considered together in an integrative approach.

This paper is organized as follows: the research background is discussed next. Then, the methodology of the research is presented in terms of three subsections: the desirability evaluation structure, the problem modeling approach, and the problem-solving algorithm. Implementation of the model is considered next in terms of a numerical example. Finally, the last section is dedicated to stating the concluding remarks.

2. Background

While many real-world problems involve analysis of more than one response variable, most of the mathematical programming applications in the literature have been focusing on single-response problems and few attempts have been made to solve multiple-objective statistical problems. We can classify these attempts into three categories (Azadivar, 1999; Coello, 2000).

The usual practice in the first category is to simplify the problem, selecting the most important response and ignoring the other responses or considering them as the model constraints. The method introduced by Coello (2000) falls into this category and is called the ε-constraint method. This method is based on minimizing one response and considering the other objectives as constraints bound by some allowable levels ε_i. Also, the procedures presented by Hartmann and Beaumont (1968) and Biles and Swain (1980) may be classified in this category. While Hartmann and Beaumont modeled the problem using a linear programming approach, Biles and Swain used this approach once in conjunction with a version of Box's complex method (Box & Draper, 1987) and alternatively along with a variation of the gradient method.

The proposed procedures of this category might generally lead to unrealistic solutions, especially when conflicting objectives are present. For example, usually in a capital investment problem with two objectives, profit maximization, and risk minimization, the higher profit led to a bigger risk. For this reason, treating this problem using a single objective will lead to a poor solution.

In the second category, some multi-attribute value functions are used. Mollaghasemi et al. (1991) used a multi-attribute value function representing the decision-maker preferences. Then, they applied a gradient search technique to find the optimum value of the assessed function. Moreover, Mollaghasemi and Evans (1994) proposed a modification of the multi-criteria mathematical programming technique called the STEP method which works interactively with the decision-maker. Teleb and Azadivar (1994) proposed an algorithm based on the constrained scalar simplex search method. This method works by calculating the objective function value in a set of vertices of a complex. It moves towards the optimum by eliminating the worst solution and replacing it with a new and better solution. The process is repeated until a convergence criterion is met. Boyle and Shin (1996) presented a method called Pair-wise Comparison Stochastic Cutting Plane (PCSCP) which combines features from interactive multi-objective mathematical programming and response surface methodology.

The third category which tries to formulate an aggregated objective function out of existing objectives includes several semi-successful attempts in the research literature (Coello, 2000). One of these attempts is called the weighted sum method and consists of adding all the objectives together using different weighting coefficients. Also, a variation of the goal programming methods falls into this category. For instance, Clayton et al. (1982), Rees et al. (1985), and Baesler and Sepúlveda (2000) used this approach along with other optimization methods. Baesler and Sepulveda integrated the goal programming and GA methods to solve the problem. Moreover, they used some statistical tests to control the random nature of the problem. Another method in this category is the goal attainment method in which in addition to the goal vector for each response, a vector of weights relating to the relative under or over the attainment of the desired goals must be elicited from the decision maker. The most serious pitfall of the methods in this category is the importance of the responses and hence the determination of the weights in the objective functions.

The review of the literature shows that in all of the three categories, especially in the third one, a search algorithm is used for optimizing the multi-response problem. Cheng et al. (2002) presented a neuro-fuzzy and GA method for optimizing multiple response problems. Schaffer (2014) introduced a new method, called the vector evaluated genetic algorithm (VEGA), which differed from the simple GA method in the way of the chromosomes' selection. Allenson (1992) used a population-based modeling on VEGA, in which gender was used to distinguish between the two objectives of a problem dealing with the planning of a route composed of several straight pipeline segments. In this method, only male-female mating is allowed and gender is randomly assigned at birth. Fourman (2014) suggested a GA-based method for lexicographic ordering problems. In his approach, the designer ranks the objectives in order of importance. The optimum solution is then obtained by optimizing the objective function, starting with the most important and proceeding according to the assigned order of importance. Périaux et al. (1997) proposed a GA-based method that uses the concept of game theory to solve a bi-objective optimization problem. Coello (1996) proposed a min-max strategy with GA. In his method, the decision maker has to provide a predefined set of weights that will be used to spawn several small subpopulations that evolve separately, each trying to converge into a single point. Fonseca and Fleming (1993) proposed a GA scheme in which the rank of an individual corresponds to the number of chromosomes in the current population by which it is dominated. Kim and Rhee (2004) proposed a method based on the desirability function and GA and applied his method to optimize a welding process. Heredia-Langner et al. (2004) presented a model—robust alphabetically—optimal design with GA. This technique is useful in situations when computer-generated designs are most likely to be employed.

3. Methodology

The methodology to be discussed here has three major parts. In the first part, a fuzzy inference system of an FES is made based on Mamdani's fuzzy inference method to make a decision support system or a control system of a real-world situation. Second, a desirability assigning structure is made to specify the fuzzy desirability set of each fuzzy response set of the first part. In this situation, the fuzzy nature of the responses makes crisp desirability functions unusable. Therefore, a customized fuzzy desirability mapping should be defined to enhance the accuracy of the answer, and this is of extreme importance in the process. Eventually, as the last step, a genetic algorithm is applied to search among the input values that optimize the whole responses simultaneously. A geometrical mean of all de-fuzzified desirability values of responses is used here as the fitness function. Also, a customized defuzzification method is used here so that crisp desirability values are gained for each response.

3.1. *Desirability evaluation structure*

The desirability function approach is one of the most widely used methods in the industry for dealing with the optimization of multiple-response problems. It is based on the idea that the quality of a product that has multiple quality characteristics is completely unacceptable if one of the characteristics lies outside the desired limits. This method assigns a score to a set of responses and chooses factor settings that maximize that score.

To describe the desirability function approach mathematically, suppose each of the k response variables is related to p independent variables by a FIS as shown in Fig. 1.

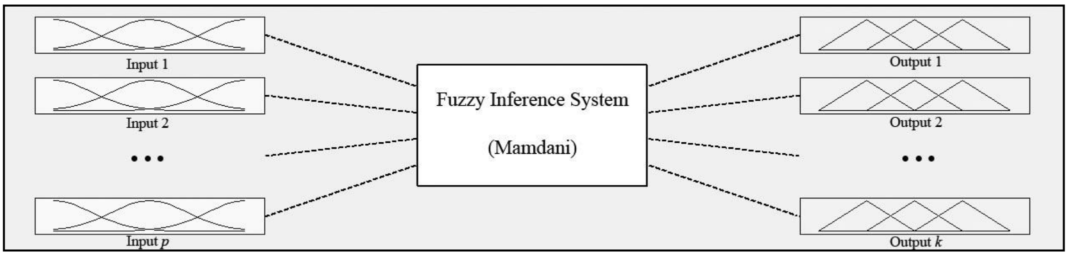

Fig. 1. A typical fuzzy inference system

If $\tilde{Y}_i$ stands for the i^{th} aggregated fuzzy output set, then a desirability function, $d_i(\tilde{Y}_i)$, assigns numbers between 0 and 1 to the possible set of each response $\tilde{Y}_i$. The value of $d_i(\tilde{Y}_i)$ increases as the desirability of the corresponding response increases. We define the overall desirability, D, by the geometrical mean of the individual desirability values shown in Eq. (1).

$$D = (d_1(\tilde{Y}_1) \times d_2(\tilde{Y}_2) \times \cdots \times d_k(\tilde{Y}_k))^{\frac{1}{k}}, \quad (1)$$

where k denotes the number of responses. Note that if a response $\tilde{Y}_i$ is completely undesirable, i.e., $d_i(\tilde{Y}_i) = 0$, then the overall desirability value is 0.

In a crisp problem, depending on whether a particular crisp response y_i is to be maximized, minimized, or assigned a target value, we can use different desirability functions.

Derringer and Suich (1980) introduced a useful class of desirability functions. There are two types of transformation from y_i to $d_i(y_i)$, namely one-sided and two-sided transformation. We employ the one-sided transformation when y_i is maximized or minimized, and the two-sided transformation when y_i is assigned to a target value.

In a two-sided transformation assume l_i and u_i are the lower and upper limits and t_i is the target value of the response y_i respectively such that $l_i < t_i < u_i$. Then we define the desirability function as Eq. (2).

$$d_i(y_i) = \begin{cases} 0, & y_i < l_i \\ \left(\frac{y_i - l_i}{t_i - l_i}\right)^s, & l_i \le y_i \le t_i \\ \left(\frac{y_i - u_i}{t_i - u_i}\right)^t, & t_i \le y_i \le u_i \\ 0, & y_i > u_i \end{cases} \quad (2)$$

Where the s and t determine how strictly the target value is desired and the user must specify their values. For $s = t = 1$ the desirability function increases linearly towards t_i, for $s < 1$ and $t < 1$ the function is convex, and for $s > 1$ and $t > 1$ the function is concave. This function for different values of s and t is graphed in Fig. 2.

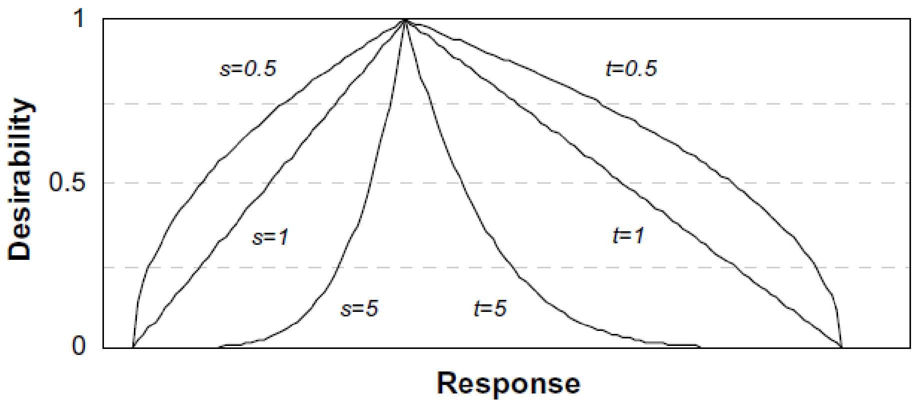

Fig. 2. Graph of the two-sided transformation

Similarly, we can define one-sided desirability functions for minimizing or maximizing cases. It should be noted that while some modified versions of the desirability functions are useful for situations where the exact mathematical methods of optimization are used, the introduced basic desirability functions are good enough for the search methods applied to the optimization problems (Del Castillo et al., 1996). For a good reference, see Derringer and Suich (1980).

According to what was discussed above, the desirability value of a crisp response is simply determined by a 1-to-1 mapping. However, this approach is not applicable when the response is fuzzy. A typical aggregated fuzzy response is shown in Fig. 3.

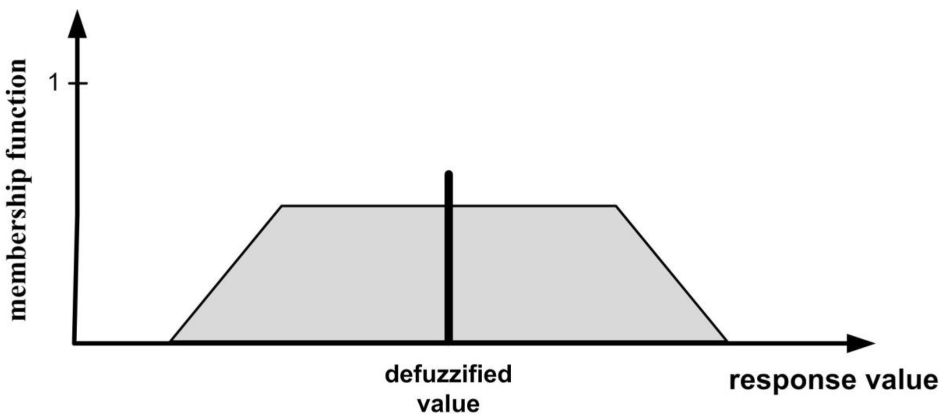

Fig. 3. A typical aggregated response value of a FIS and its de-fuzzified value

At first sight, it may be concluded that the desirability value could be obtained directly by considering the de-fuzzified value of the fuzzy response. Suppose that there are two different desirability functions defined for this response, the ones shown in Fig. 4. It is quite obvious that the desirability value reaches amount 1 in Fig. 4 (B) at a point of the aggregated fuzzy response set with the maximum degree of membership, and it means that the desirability of the response could not be equal to zero. However, if we use the de-fuzzified value for the desirability of the response, this value would be 1 for Fig. 4 (A) and zero for Fig. 4 (B), and this could not be realistic.

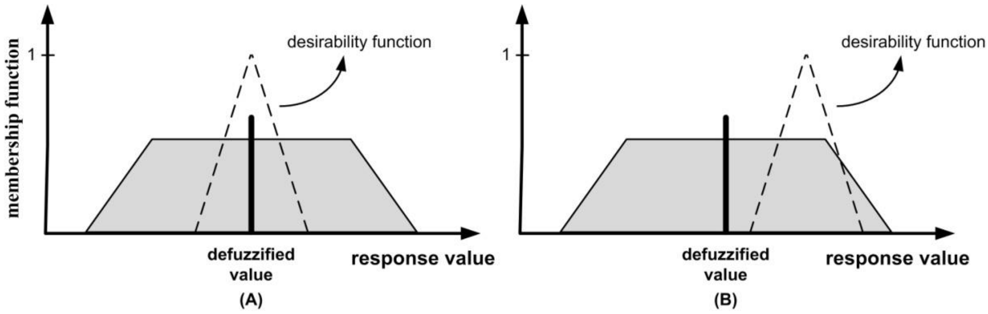

Fig. 4. A typical aggregated response with two different desirability functions

This problem made us define a new method to evaluate the desirability function for a fuzzy set. Simply, if we want to allocate a desirability value for each member of a fuzzy response set, it should be a positive number at any point which has a nonzero degree of membership and desirability value and should gain zero amounts at the points with one of these two values equal to zero. So, a fast conclusion is that the point-to-point product of the degree of membership and desirability values could represent a good desirability for each point of the domain of the definition of the response. By this definition, the justified graph of the fuzzy desirability sets for the responses displayed in Fig. 4 would be illustrated in Fig. 5.

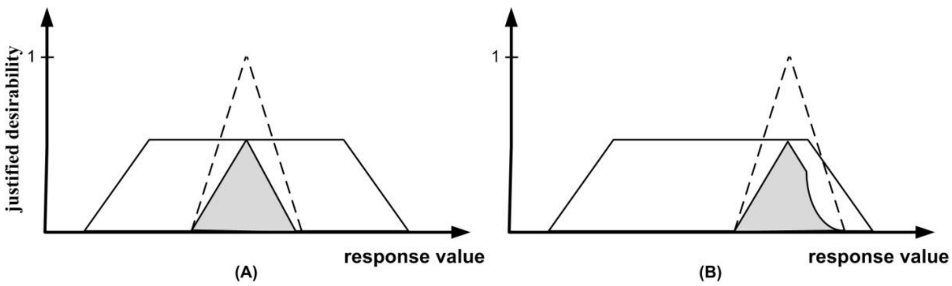

Fig. 5. Justified desirability graphs of Fig. 4

Still, this definition is not a good representative of the final individual desirability. To explain the reason for this claim, another example is prepared in Fig. 6. As it is displayed in these graphs, there are two different fuzzy sets with the same desirability function. If we consider the point-to-point product as the final desirability, the answer would be equal for both, but the desirability set should not be the same in such situations.

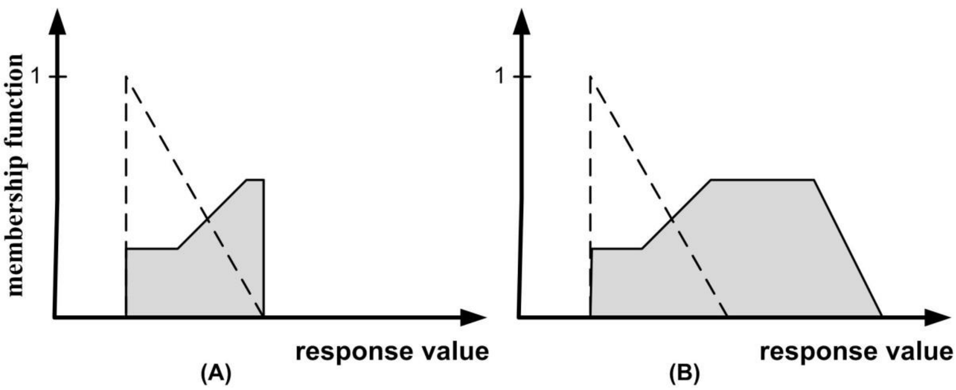

Fig. 6. Two different fuzzy sets with the same desirability function

Accordingly, we define the final individual desirability as the multiplication of the previous desirability by a proportion of the surface area under the fuzzy response graph which has nonzero desirability. This proportion would be a real number and decreases the levels of the previous desirability values by a constant amount. For example, suppose that S_A is the surface area under the highlighted graph of Fig. 6 (A) and S_B of Fig. 6 (B). Hence, the previous desirability (obtained from the point-to-point product formulation) must be multiplied by 1 in the graph (A) and by S_A/S_B in the graph (B). The result is displayed in Fig. 7 (the highlighted area).

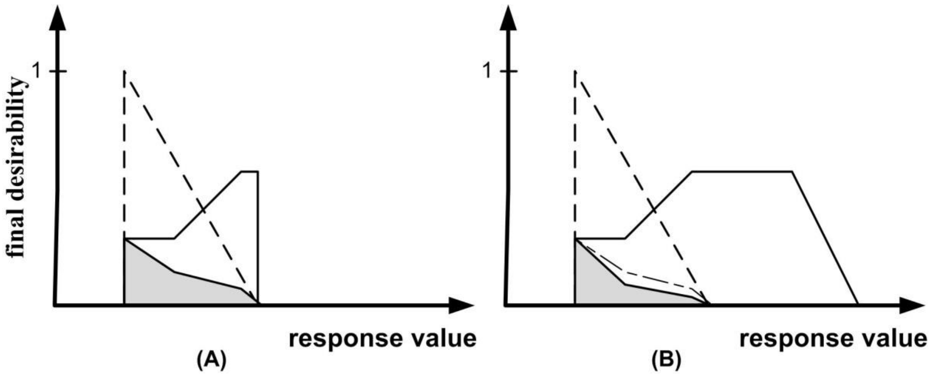

Fig. 7. The final fuzzy desirability sets for two fuzzy sets

Thus, the final individual desirability sets are in the form of a fuzzy set for each response. Nevertheless, to calculate the overall desirability of all responses by the geometrical mean formula, as explained previously, a de-fuzzified desirability value is needed for each response. We define this value by the proportion of the surface area of the final desirability set to the surface area of the initial desirability function, which would be a real number between 0 and 1.

Example 1. A fuzzy inference system according to Mamdani's method with two response variables is applied to control a system, and the aggregated fuzzy responses of this FIS for some input variables are evaluated as explained below:

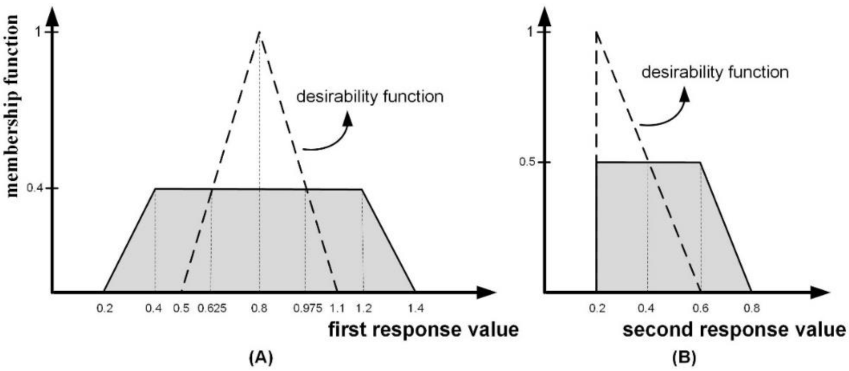

Fig. 8. The aggregated responses of an imaginary FIS with their desirability functions

The de-fuzzified desirability value for each response is calculated by the following relations:

First response:

The surface area of the response set (the highlighted trapezius shape in Fig. 8-A) $= \frac{1}{2}(1.2 + 0.8)(0.4) = 0.4$

The surface area of the response with nonzero desirability amount (the highlighted rectangle shape in Fig. 9-A) $= (0.6)(0.4) = 0.24$

The surface area of the desirability function (the triangle shape in Fig. 8-A) $= \frac{1}{2}(0.6)(1) = 0.3$

The surface area of the final desirability set (the highlighted triangle shape in Fig. 10-A) $= \frac{1}{2}\big((0.4)(0.24/0.4)\big)(0.6) = 0.072$

The de-fuzzified desirability of the response $= \frac{0.072}{0.3} = 0.24$

Second response:

The surface area of the response set (the highlighted trapezius shape in Fig. 8-B) $= \frac{1}{2}(0.6 + 0.4)(0.5) = 0.25$

The surface area of the response with nonzero desirability amount (the highlighted rectangle shape in Fig. 9-B) $= (0.4)(0.5) = 0.2$

The surface area of the desirability function (the triangle shape in Fig. 8-B) $= \frac{1}{2}(0.4)(1) = 0.2$

The surface area of the final desirability set (the highlighted triangle shape in Fig. 10-B) $= \frac{1}{2}\left((0.5)(0.2/0.25)\right)(0.4) = 0.08$

The de-fuzzified desirability of the response $= \frac{0.08}{0.2} = 0.4$

Overall desirability $= \sqrt[2]{(0.24)(0.4)} = 0.310$ (according to Eq. 1)

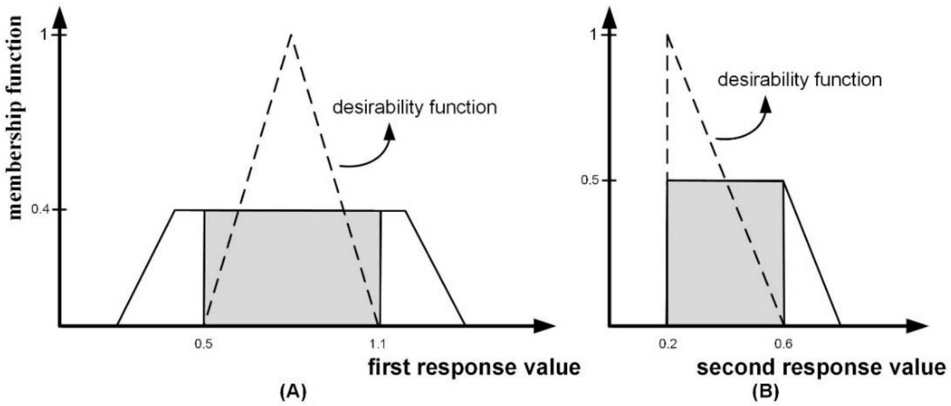

Fig. 9. Areas with nonzero desirability values

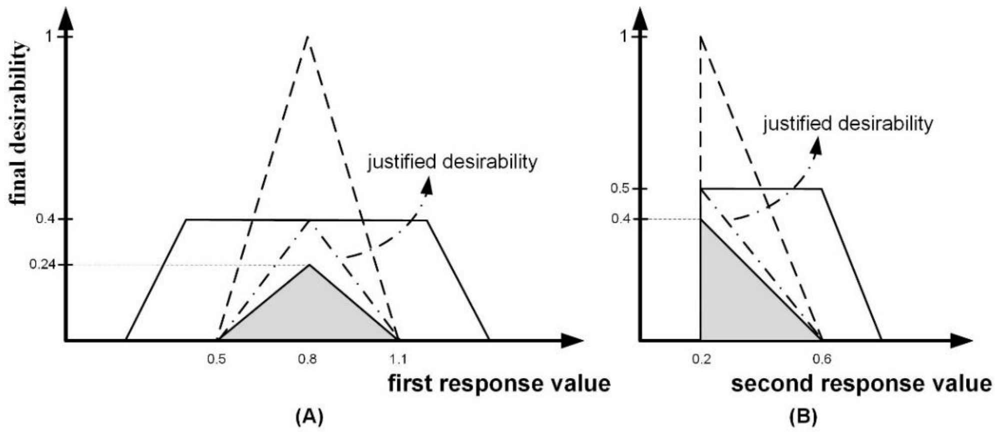

Fig. 10. Diagrams of the final desirability sets

The surfaces of areas with nonzero desirability values for both of the responses are displayed in Fig. 9. Also, the diagrams of the final individual desirability sets are displayed in Fig. 10.■

Example 2. By this definition of desirability for a fuzzy set, the final desirability set for some examples is indicated in Fig. 11. As expected, the de-fuzzified desirability value is equal to 1 in Graph (B) ($d_2(\tilde{B}) = 1$), but it is less than 1 in Graph (A) ($d_1(\tilde{A}) < 1$).

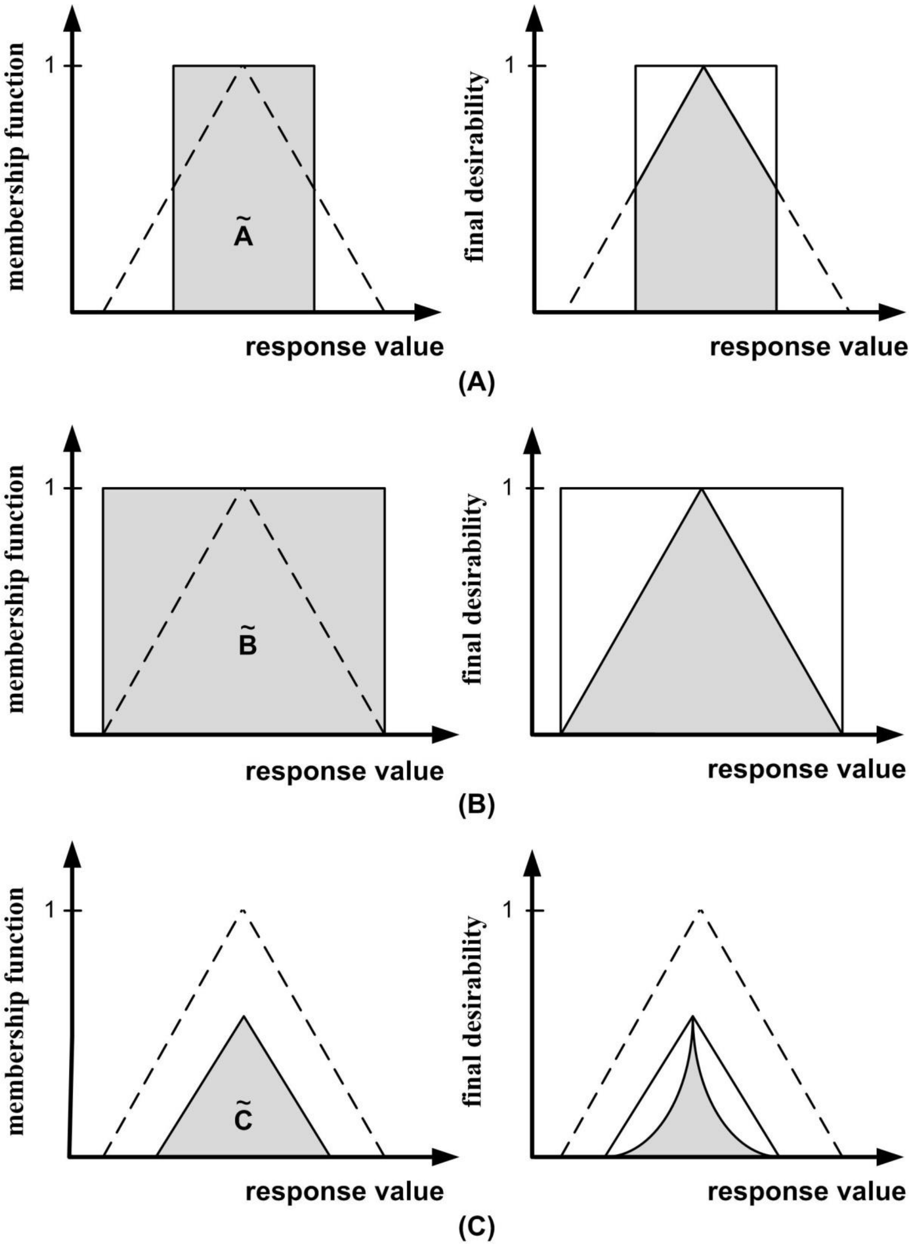

Fig. 11. Typical fuzzy sets with their desirability functions and final desirability sets

Although the degree of membership is equal to 1 for all of the members of $\tilde{A}$, it becomes zero at some points with nonzero desirability values. Whereas any member of $\tilde{B}$ has nonzero but equal to 1 degree of membership at any point with nonzero desirability value, and this makes its de-fuzzified desirability become 1. In addition, the de-fuzzified desirability value of $\tilde{C}$ is less than 1 ($d_3(\tilde{C}) < 1$). It should be considered that the de-fuzzified desirability value of a fuzzy set reaches the amount 1 if and only if its degree of membership is 1 at any point with a nonzero desirability value and zero at other points. The mathematical expression of this statement can be presented by the following relations for an arbitrary fuzzy set $\tilde{Y}$:

$$d(\tilde{Y}) = 1 \quad \text{IFF} \quad \forall y \in U, (d(y) \neq 0 \Longrightarrow \mu_{\tilde{Y}}(y) = 1) \,\&\, (d(y) = 0 \Longrightarrow \mu_{\tilde{Y}}(y) = 0)$$

where U stands for the universal discourse and here denotes the definition domain of the response. ■

It should be considered that we define the final desirability value for fuzzy singletons to be equal to the multiplication of the degree of membership of the singleton by the desirability value of that point. Thus, this definition of fuzzy desirability function stands for a realistic generalization of the desirability functions of crisp values.

3.2. *Problem modeling*

Knowing that a search algorithm such as GA needs scalar fitness information to work, the simplest idea to model a multi-response statistical optimization problem is to combine all responses into a single one within the desirability function framework. Desirability functions have many advantages in comparison to other combining techniques since they have a very flexible role. It means that we may maximize some of the responses and minimize others and set target values for some of them simultaneously.

For modeling the problem, the following information is primarily provided by decision-makers:

1. All the factors that make up the input of the problem. These factors are the independent variables $x_1, \ldots, x_p$.
2. The lower and upper bounds of the independent variables ($L(x_h)$ and $U(x_h)$).
3. The output of the problem. This output is the fuzzy response sets denoted by $\tilde{Y}_1, \ldots, \tilde{Y}_k$.
4. One-sided or two-sided desirability functions for each response. A one-sided or two-sided transformation for each response depends on the nature of the objective of the problem.

Then the mathematical model of the problem becomes:

$$\begin{aligned} \max \quad & D = \sqrt[k]{\left(d_1(\tilde{Y}_1) \times d_2(\tilde{Y}_2) \times \cdots \times d_k(\tilde{Y}_k)\right)} \\ \text{s.t.} \quad & L(x_h) \le x_h \le U(x_h), \\ & h = 1, 2, \ldots, p. \end{aligned} \tag{3}$$

In model (3), the objective function is the aggregation of de-fuzzified desirability values of different responses using the geometrical mean based on Eq. (1) and Section 4 discussions. Additionally, the lower and upper bounds of input variables are specified by decision-makers.

3.3. *The problem-solving algorithm*

The model stated in Eq. (3) is based on the fuzzy inference system (Mamdani's method). The objective function is generated here by maximizing the desirability values of fuzzy responses. A heuristic search algorithm is applicable here to solve the model considering its non-relational and complex nature. The outstanding optimization capability of GA in the presence of aggregated function of multiple objectives Coello (2000) was the reason to the candidate it here.

A common formulation of GA is described in Golberg (1989). Natural selection and the initiation mechanism of employing random solutions (entitled population) are the pillars of this stochastic search algorithm. This makes GA distinguishable from conventional search algorithms. Individuals of the population, known as chromosomes, are indeed the solutions to the problem. Chromosomes evolve in successive iterations of the algorithm known as generations. The evaluation of chromosomes is performed via the measure of fitness, i.e., the aggregated objective function of Eq. (3). Crossover or mutation operators are employed to generate new chromosomes, entitled offspring, in the process of establishing the next generation. The evaluation of chromosomes is the key action in this process. Accordingly, the best chromosome appears by the convergence of GA after some generations.

The computer program of the model is developed here using MATLAB and it consists of a FIS and two sub-functions. The main program asks users to enter the necessary information. Then it calls the genetic algorithm toolbox to solve the problem, considering the overall desirability function for the fitness function. The overall desirability of each set of inputs is calculated in the first sub-function by calling the second sub-function repeatedly. The second sub-function serves to compute the individual desirability values according to the discussions in Section 3.1.

4. Implementation of the model

An application of the presented optimization model will be presented in this part in a numerical example of a multi-input multi-output (MIMO) fuzzy inference system. Searching among the input variables is performed here to maximize the overall desirability. The indicated search algorithm, i.e., the genetic algorithm, serves to find the optimum input levels resulting in the desired output values. Since fuzzy inputs and outputs are considered here, the proposed desirability evaluation structure of fuzzy variables is employed. Fuzzy outputs of the implemented model are specified based on fuzzy inputs according to rule-based instructions. Clarifying graphs about the desirability evaluation structure and the algorithm convergence process in successive generations are provided here.

Numerical Example: *Client Asset Allocation Model*

The inputs (linguistic variables) in the fuzzy logic client asset allocation model are age and risk tolerance (risk). It is important to observe that there are three outputs (linguistic variables) here, savings, income, and growth. Hence this is a two-input-three-output model.

The control objective is for any given pair (age, risk) which reflects the state of a client to find how to allocate the asset to savings, income, and growth.

Assume that the financial experts describe the two input and three output variables using triangular and trapezoidal shapes as follows:

Age (input 1) ≜ {**Y**(*young*), **MI**(*middle age*), **OL**(*old*)},
Risk (input 2) ≜ {**L**(*low*), **MO**(*moderate*), **H**(*high*)},
Saving (output 1) ≜ {**L**(*low*), **M**(*medium*), **H**(*high*)},
Income (output 2) ≜ {**L**(*low*), **M**(*medium*), **H**(*high*)},
Growth (output 3) ≜ {**L**(*low*), **M**(*medium*), **H**(*high*)}.

They are shown in Figs.12-14.

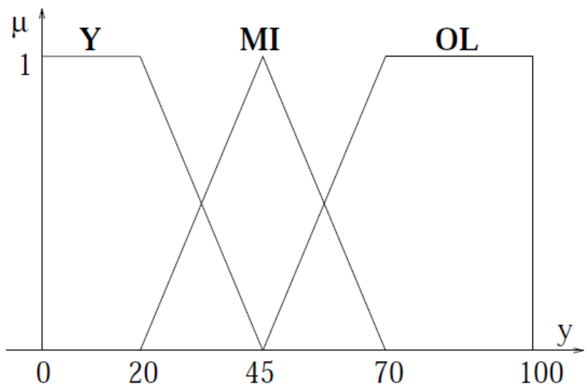

Fig. 12. Terms of the input *age*

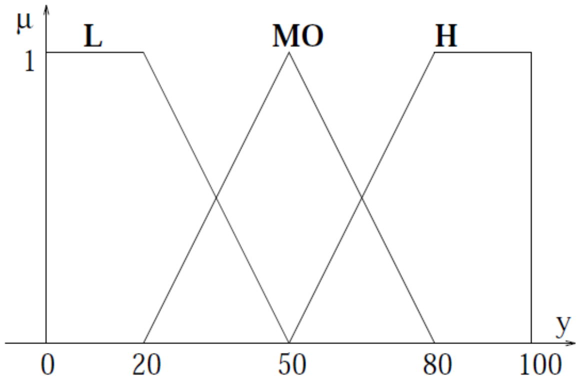

Fig. 13. Terms of the input *risk tolerance*

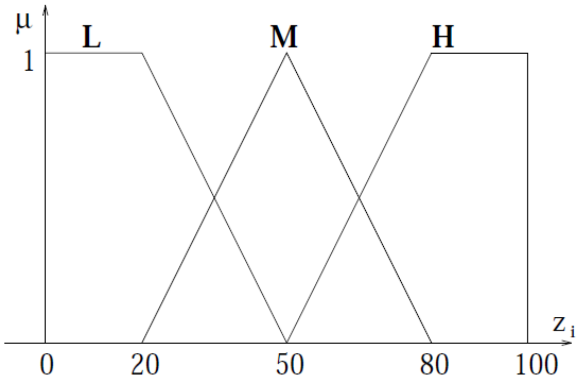

Fig. 14. Terms of the output variables *savings*, *income*, *growth*

The universal sets (operating domains) of the input and output variables are $U_1 = \{x|0 \leq x \leq 100\}$ where the base variable x represents years, $U_2 = \{y|0 \leq y \leq 100\}$ with base variable y measured on a psychometric scale, $U_3 = \{z_i|0 \leq z_i \leq 100, i = 1, 2, 3\}$ where the base variables z_i take values on a scale from 0 to 100.

The variable *age* (Fig. 12) differs slightly from the other variables; the membership functions of its terms are:

$$\mu_Y(x) = \begin{cases} 1 & x \leq 20 \\ \dfrac{45 - x}{25} & 20 \leq x \leq 45 \end{cases}$$

$$\mu_{MI}(x) = \begin{cases} \dfrac{x - 20}{25} & 20 \leq x \leq 45 \\ \dfrac{70 - x}{25} & 45 \leq x \leq 70 \end{cases} \tag{4}$$

$$\mu_{OL}(x) = \begin{cases} \dfrac{x - 45}{25} & 45 \leq x \leq 70 \\ 1 & 70 \leq x \end{cases}$$

The terms of linguistic variables *risk*, *savings*, *income*, and *growth* are displayed in Figs. 13 and 14; their membership functions of them are:

$$\mu_L(v) = \begin{cases} 1 & 0 \leq v \leq 20 \\ \dfrac{50 - v}{30} & 20 \leq v \leq 50 \end{cases} \tag{5}$$

$$\mu_M(v) = \mu_{MO}(v) = \begin{cases} \dfrac{v-20}{30} & 20 \leq v \leq 50 \\ \dfrac{80-v}{30} & 50 \leq v \leq 80 \end{cases}$$

$$\mu_H(v) = \begin{cases} \dfrac{v-50}{30} & 50 \leq v \leq 80 \\ 1 & 80 \leq v \leq 100 \end{cases}$$

There are nine *if ... and ... then* rules, and each inference rule produces three conclusions, one for *savings*, one for *income*, and one for *growth*. Consequently, the financial experts have to design three decision tables. Assume that these are the tables presented below.

Table 1. Decision table for the output *savings*

		Risk tolerance		
		Low	Moderate	High
Age	Young	M	L	L
	Middle	M	L	L
	Old	H	M	M

Table 2. Decision table for the output *income*

		Risk tolerance		
		Low	Moderate	High
Age	Young	M	M	L
	Middle	H	H	M
	Old	H	H	M

Table 3. Decision table for the output *growth*

		Risk tolerance		
		Low	Moderate	High
Age	Young	M	H	H
	Middle	L	M	H
	Old	L	L	M

For instance, the first two *if ... then* rules read:

If the client's age is young and the client's risk tolerance is low, then asset allocation is medium in savings, medium in income, and medium in growth.

If the client's age is young and the client's risk tolerance is moderate, then asset allocation is low in savings, medium in income, and high in growth.

Consider a client whose age is $x_0 = 25$ and whose risk tolerance level is $y_0 = 45$. Matching the readings 25 and 45 against the appropriate terms in Figs. 12 and 13 and using Eqs. (4) and (5) give the fuzzy reading inputs:

$$\mu_Y(25) = \tfrac{4}{5},\ \mu_{MI}(25) = \tfrac{1}{5},\ \mu_L(45) = \tfrac{1}{6},\ \mu_{MO}(45) = \tfrac{5}{6}.$$

The strength of the rules calculated using Mamdani's fuzzy inference method is:

$$\alpha_{11} = \mu_Y(25) \wedge \mu_L(45) = \min\left(\tfrac{4}{5}, \tfrac{1}{6}\right) = \tfrac{1}{6},$$

$$\alpha_{12} = \mu_Y(25) \wedge \mu_{MO}(45) = \min\left(\frac{4}{5}, \frac{5}{6}\right) = \frac{4}{5},$$

$$\alpha_{21} = \mu_{MI}(25) \wedge \mu_L(45) = \min\left(\frac{1}{5}, \frac{1}{6}\right) = \frac{1}{6},$$

$$\alpha_{22} = \mu_{MI}(25) \wedge \mu_{MO}(45) = \min\left(\frac{1}{5}, \frac{5}{6}\right) = \frac{1}{5}.$$

The control outputs of the rules are presented in the active cells in three decision tables.

Table 4. Control output *savings*

	Low	Moderate
Young	$\frac{1}{6} \wedge \mu_M(z_1)$	$\frac{4}{5} \wedge \mu_L(z_1)$
Middle	$\frac{1}{6} \wedge \mu_M(z_1)$	$\frac{4}{5} \wedge \mu_L(z_1)$

Table 5. Control output *income*

	Low	Moderate
Young	$\frac{1}{6} \wedge \mu_M(z_2)$	$\frac{4}{5} \wedge \mu_M(z_2)$
Middle	$\frac{1}{6} \wedge \mu_H(z_2)$	$\frac{1}{5} \wedge \mu_H(z_2)$

Table 6. Control output *growth*

	Low	Moderate
Young	$\frac{1}{6} \wedge \mu_M(z_3)$	$\frac{4}{5} \wedge \mu_H(z_3)$
Middle	$\frac{1}{6} \wedge \mu_L(z_3)$	$\frac{1}{5} \wedge \mu_M(z_3)$

The outputs in the four active cells in Tables 4-6 have to be aggregated separately. The results (see Figs. 15-17) are:

$$\mu_{agg}(z_1) = \max\left\{\min\left(\frac{1}{6}, \mu_M(z_1)\right), \min\left(\frac{4}{5}, \mu_L(z_1)\right)\right\};$$

$$\mu_{agg}(z_2) = \max\left\{\min\left(\frac{4}{5}, \mu_M(z_2)\right), \min\left(\frac{1}{5}, \mu_H(z_2)\right)\right\};$$

$$\mu_{agg}(z_3) = \max\left\{\min\left(\frac{1}{5}, \mu_M(z_3)\right), \min\left(\frac{4}{5}, \mu_H(z_3)\right), \min\left(\frac{1}{6}, \mu_L(z_3)\right)\right\}.$$

The aggregated outputs are shown in Figs. 15-17 are de-fuzzified by using the *Height defuzzification method* (HDM). The results are given in the same figures.

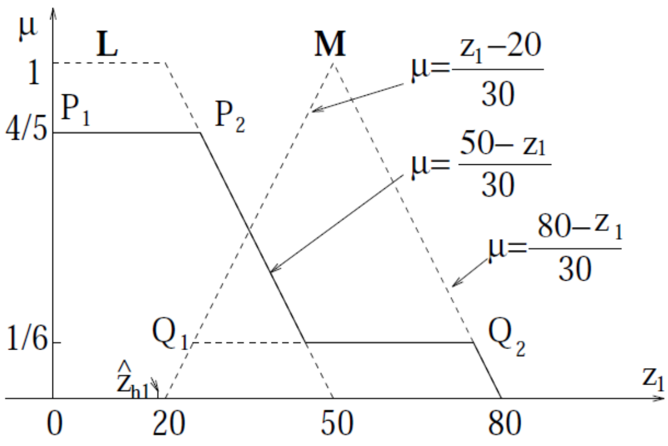

Fig. 15. Aggregated output *savings* and their defuzzification

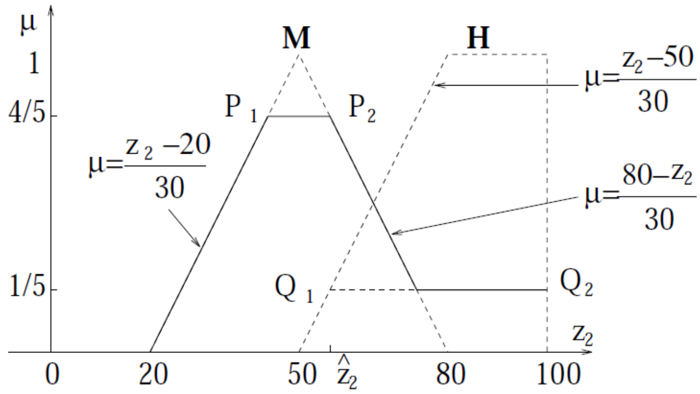

Fig. 16. Aggregated output *income* and its defuzzification

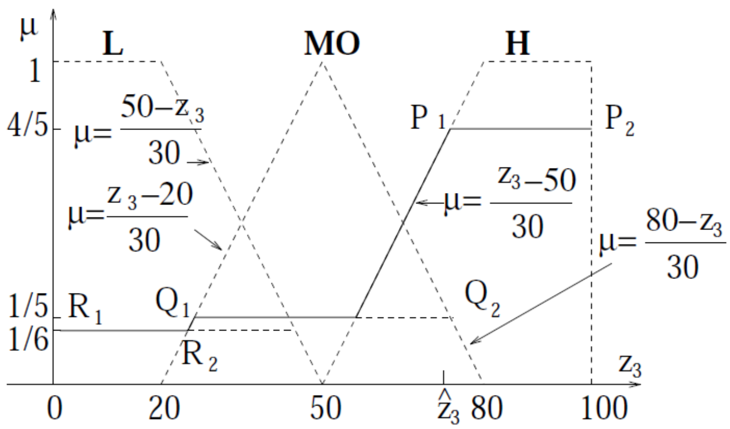

Fig. 17. Aggregated output *growth* and its defuzzification

The projections of the flat segments can be easily found using their height and the relevant equations of inclined segments indicated in the figures. For instance, consider Fig. 15. Substituting $\frac{4}{5}$ for μ in $\mu = \frac{50-z_1}{30}$ gives the projection of P_2 to be 26. Substituting $\frac{1}{6}$ for μ in $\mu = \frac{z_1-20}{30}$ and $\mu = \frac{80-z_1}{30}$ gives the projection of Q_1 and Q_2 to be 25 and 75. Similarly one can find that the projections of P_1P_2 and Q_1Q_2 in Fig. 16 are the intervals [44,56] and [56,100]. There are three flat segments P_1P_2, Q_1Q_2, and R_1R_2 in Fig. 17. Their projections are [74,100], [26,74], and [0,45].

Then using the HDM defuzzification formula we find

$$\hat{z}_{h1} = \frac{\frac{4}{5}\left(\frac{0+26}{2}\right) + \frac{1}{6}\left(\frac{25+75}{2}\right)}{\frac{4}{5}+\frac{1}{6}} = 19.38\ (savings)$$

$$\hat{z}_{h2} = \frac{\frac{4}{5}\left(\frac{44+56}{2}\right) + \frac{1}{5}\left(\frac{56+100}{2}\right)}{\frac{4}{5}+\frac{1}{5}} = 55.60\ (income)$$

$$\hat{z}_{h3} = \frac{\frac{4}{5}\left(\frac{74+100}{2}\right) + \frac{1}{5}\left(\frac{26+74}{2}\right) + \frac{1}{6}\left(\frac{0+45}{2}\right)}{\frac{4}{5}+\frac{1}{5}+\frac{1}{6}} = 71.44\ (growth)$$

Now suppose that our adopted desirability function for the output of *savings* is the triangular fuzzy number [0,0,25]; also assume [20,50,80] and [75,100,100] to be the corresponding functions of outputs *income* and *growth*. The desirability functions aggregated outputs, and final desirability functions are plotted together in Figs. 18-20. The de-fuzzified desirability values are:

d(*savings*) = 0.4571
d(*income*) = 0.5556
d(*growth*) = 0.3954
Overall desirability = $\sqrt[3]{0.4571 \times 0.5556 \times 0.3954}$ = 0.4648

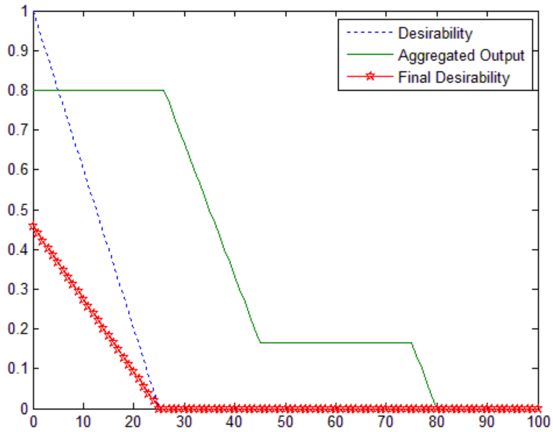

Fig 18. The aggregated output, desirability, and final desirability functions graphs of output *savings* for inputs *age* = 25, *risk* = 45

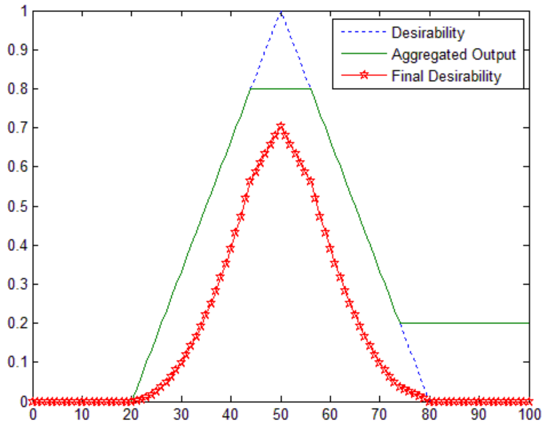

Fig 19. The aggregated output, desirability, and final desirability functions graphs of output *income* for inputs *age* = 25, *risk* = 45

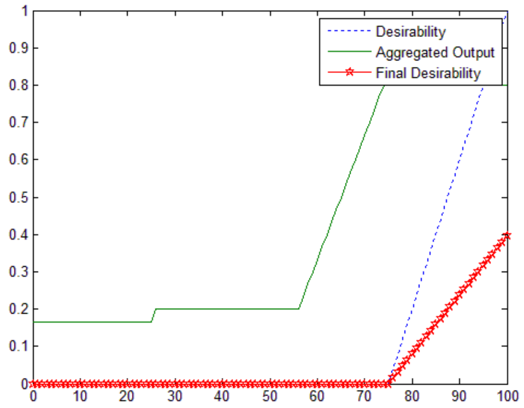

Fig 20. The aggregated output, desirability, and final desirability functions graphs of output *growth* for inputs *age* = 25, *risk* = 45

It is clear that if the de-fuzzified value for output *growth* ($\hat{z}_{h3} = 71.44$) had been used to determine its desirability, the *growth* individual desirability of the case would have become zero, and this makes the overall desirability become zero too, which does not make sense considering Fig. 20.

The optimized input, output, and desirability values for this adopted desirability functions of the responses are obtained via this model as follows:

Suggested input values:

$age^* = 15.9363$
$risk^* = 50.0156$

Optimum de-fuzzified responses:

$savings^* = 18.3077$
$income^* = 49.9854$
$growth^* = 81.6923$

The maximized overall desirability = 0.689118

The diagram of the corresponding GA search for the adopted desirability functions is displayed in Fig. 21. The related aggregated responses and their adopted and final desirability diagrams are shown in Figs. 22-24.

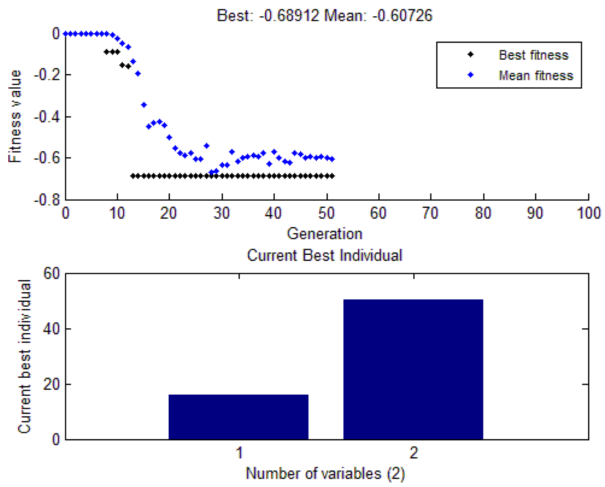

Fig. 21. The results of genetic algorithm optimization for the *Client Asset Allocation Model* of FIS for the adopted desirability functions

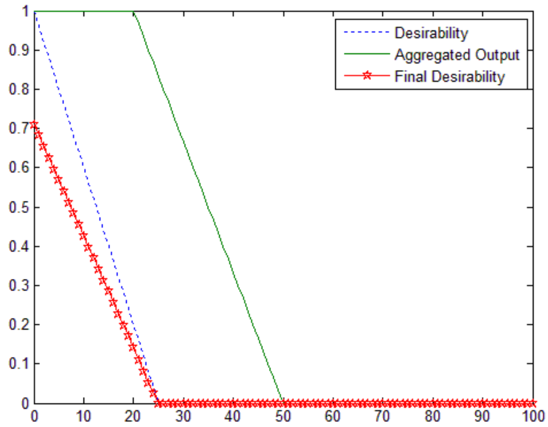

Fig 22. The optimum aggregated output adopted desirability and final desirability functions graphs of output *savings* for inputs $age^* = 15.9363, risk^* = 50.0156$

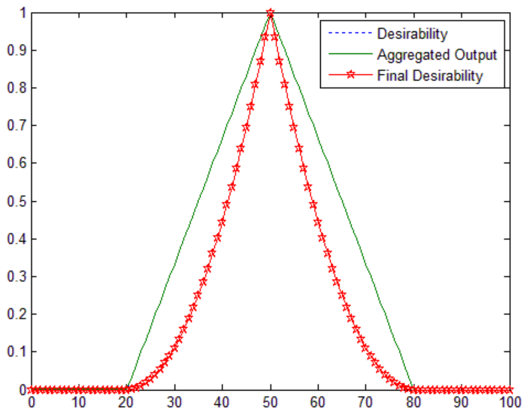

Fig 23. The optimum aggregated output adopted desirability and final desirability functions graphs of output *income* for inputs $age^* = 15.9363, risk^* = 50.0156$

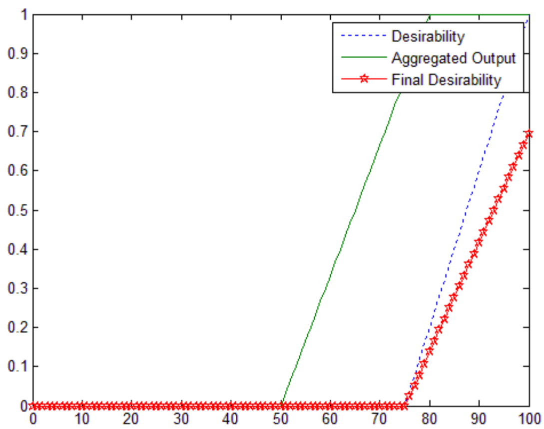

Fig 24. The optimum aggregated output adopted desirability and final desirability functions graphs of output *growth* for inputs $age^* = 15.9363, risk^* = 50.0156$

The corresponding de-fuzzified desirability values which lead to the maximized overall desirability value are:

d(*savings*) = 0.7082 d(*income*) = 0. 6668 d(*growth*) = 0. 6930

5. Conclusion

Nowadays the necessity of the optimization process and its application in different areas of engineering techniques is crystal clear, and plenty of various methods which accomplish this mission are available now. It plays an important role in techniques such as Multi-Criteria Decision Making (MCDM), systems' simulated models, and also mathematical programming models. The optimization method implemented on fuzzy inference systems in this article could be applied to different cases the same as the one discussed in the present article. As an application of the case study, the administrative board of a company may decide to employ a manager who is the best candidate to achieve the firm's desirability criteria to minimize the *savings* of the firm, maximize its *growth*, and maintain the income at a moderate *level*. The FIS presented in the case study is a mapping that inputs the *age* and *risk tolerance* (that could be determined by another FIS) of a person as the manager of a company and forecasts the potential *savings*, *income*, and *growth* that could be achieved in the company. Therefore, our model excludes the optimum inputs which satisfy the administrative board's desirability criteria in its best condition.

As the prospective opportunities for future research, the following considerations are helpful. The current research has considered a simple fuzzy inference system as the expert system for emulating the reasoning process of human experts; however, more developed expert systems are introduced in the literature which is applicable here. Artificial intelligence, machine learning, and other data-driven systems are among these methods. Moreover, the investigation of desirability evaluation structure for other types of fuzzy variables is another open problem related to the present study.

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